Homework: http://www..math.mnrt.net

Name ______________________ ___ raw ___ scaled ___ percent

Math 10 Review

Circles

■ **Partial credit may be given for correct work. Therefore, it is to your advantage to write clear solutions. If I cannot understand a solution within 60 seconds, then it will receive no partial credit.**

■ **A. Answer the following. (2 points each). This section is 4% of exam.**

[1] Write the equation in center-radius form of a circle centered at the origin having radius *5*.

$x^2 + y^2 = 25^2$

[2] Write the equation in center-radius form of a circle centered at $C(2, -3)$ radius *4*.

$(x-2)^2 + (y+3)^2 = 16$

[3] Write the equation of the line tangent to a circle of radius r at point (x_0, y_0) on the circumference.

$x_0 x + y_0 y = r^2$

■ **B. Answer the following. (28 points each). This section is 92% of exam.**

[1] Find the equation of the line tangent to the circle $x^2 + y^2 = 20$ at the point $(-4, 2)$.

Since $(-4, 2)$ is on the circle, $(-4, 2)$ is (x_0, y_0).

Then,

$-4x + 2y = 20$

[2] Find the equations of the lines through (−2, 1)that are tangent to the circle $x^2 + y^2 = 1$.

FIRST MAKE A SKETCH! ALWAYS!

$x_0\, x + y_0\, y = r^2$ is the equation of *any* line tangent to a circle at (x_0, y_0). All you have to do is determine the constants x_0 and y_0.

You are after two unknowns, so you will need two equations in those unknowns; i.e. simultaneous equations in x_0 and y_0.

Here's how you get them.

You know that in $x_0\, x + y_0\, y = r^2$, x_0 and y_0 are on the circle. Thus, one equation is

EQ1 $x_0^2 + y_0^2 = 1$

You know that the point (−2, 1) is on each line. So,

EQ2 $-2\, x_0 + y_0 = 1$ {This is $x_0\, x + y_0\, y = r^2$ with $x = -2,\ y = 1,\ r = 1$}

So you solve

$$\begin{pmatrix} x_0^2 + y_0^2 & = & 1 \\ -2\, x_0 + y_0 & = & 1 \end{pmatrix}$$

Finding that $\boxed{\begin{matrix} x_0 = \frac{-4}{5} \\ y_0 = \frac{-3}{5} \end{matrix}}$ or $\boxed{\begin{matrix} x_0 = 0 \\ y_0 = 1 \end{matrix}}$.

Now you know the constants in $x_0\, x + y_0\, y = r^2$.

Therefore, one line is $\frac{-4}{5}\, x - \frac{3}{5}\, y = 1$ and the other line is $y = 1$.

[3] Find the equations of the lines of slope 2 that are tangent to the circle $x^2 + y^2 = 1$.

FIRST MAKE A SKETCH! ALWAYS!

$x_0\, x + y_0\, y = r^2$ is the equation of *any* line tangent to a circle at (x_0, y_0). All you have to do is determine the constants x_0 and y_0.

You are after two unknowns, so you will need two equations in those unknowns; i.e. simultaneous equations in x_0 and y_0.

Here's how you get them.

You know that in $x_0\, x + y_0\, y = r^2$, x_0 and y_0 are on the circle. Thus, one equation is

EQ1 $x_0^2 + y_0^2 = 1$

Now, you must get another equation in x_0 and y_0.

You know that the tangent line is **perpendicular** to the circle at the point of tangency. And, you know that the slopes of perpendicular lines are related by $m_1\, m_2 = -1$.

The slope of the tangent line was given. It is $m_{\tan} = 2$. Thus,

$m_{\tan}\, m_{\text{radius}} = -1$

$2\, m_{\text{radius}} = -1$

Since thye slope of the radius at (x_0, y_0) is $\frac{y_0}{x_0}$ {isn't the sketch helpful?)

$2\left(\frac{y_0}{x_0}\right) = -1$

Whence,

EQ2 $y_0 = \frac{-1}{2}\, x_0$

So you solve

$$\begin{pmatrix} x_0^2 + y_0^2 & = & 1 \\ y_0 & = & \frac{-1}{2}\, x_0 \end{pmatrix}$$

Finding that $\boxed{\begin{array}{l} x_0 = -\frac{2\sqrt{5}}{5} \\ y_0 = \frac{\sqrt{5}}{5} \end{array}}$ or $\boxed{\begin{array}{l} x_0 = \frac{2\sqrt{5}}{5} \\ y_0 = -\frac{\sqrt{5}}{5} \end{array}}$.

Now you know the constants in $x_0\, x + y_0\, y = r^2$.

Therefore, one line is $-\frac{2\sqrt{5}}{5}\, x - \frac{\sqrt{5}}{5}\, y = 1$ and the other line is .$\frac{2\sqrt{5}}{5}\, x - \frac{\sqrt{5}}{5}\, y = 1$

[4] At how many points does the circle $x^2 + y^2 - 4x + 6y + 9 = 0$

[a] intersect the x-axis? ***Answer:*** ________0______

[b] intersect the y-axis? ***Answer:*** ________1______

Method I

Rewrite in center-radius form

$(x-2)^2 + (y+3)^2 = 4$

When you graph this the answers to the above questions will be obvious.

Method 2

If the circle crosses the x-axis, y will be zero at the point of intersection. Setting $y = 0$ in $x^2 + y^2 - 4x + 6y + 9 = 0$ yields $x^2 - 4x + 9 = 0$. You are not interested in the solutions themselves, but only how many there are. Evaluating the discriminate of this quadratic shows $D = -20$. So the circle does *not* touch the x-axis.

If the circle crosses the y-axis, x will be zero at the point of intersection. Setting $x = 0$ in $x^2 + y^2 - 4x + 6y + 9 = 0$ yields $y^2 + 6y + 9 = 0$. You are not interested in the solutions themselves, but only how many there are. Evaluating the discriminate of this quadratic shows $D = 0$. So the circle touches the y-axis exactly once.

[5] Find the center and the radius of the circle with equation $x^2 + y^2 - x + 4y + 2 = 0$.

$\left(x - \frac{1}{2}\right)^2 + (y+2)^2 = \frac{5}{2}$

So, center at $\left(\frac{1}{2}, -2\right)$, $r = \sqrt{\frac{5}{2}}$.

■ **C. Answer the following. (6 points). This section is 4% of exam.**

[1] A point $P(x, y)$ traces a path such that its distance from a fixed point $F(0, 2)$ equals its distance to the line $y = -2$.

[a] Find the equation for the path point P traces. *Answer:* _____ $y = \frac{1}{8} x^2$ ________

[b] The path point P traces is what figure? *Answer:* ______ **Parabola** _______

Make a sketch. Without one, this is a difficult problem.

Note: There are questions that may appear quite different from these that I can ask to find out the degree to which you understand this material. So, when you work these, strive to understand precisely why each step is taken.